Werbellinsee Team-Competition 2012

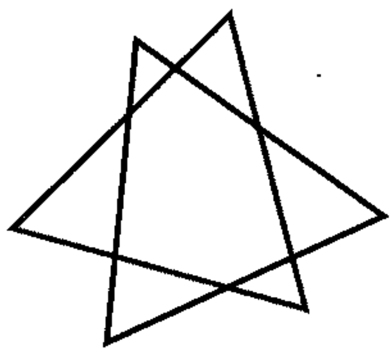
1. Consider a standard 8×8 chessboard consisting of 64 small squares coloured in the usual pattern, so 32 are black and 32 are white. A zig-zag path across the board is a collection of eight white squares, one in each row, which meet at their corners. How many zig-zag paths are there?

2. Each of Paul and Jenny has a whole number of euros.
   He says to her: 'If you give me €3, I will have n times as much as you'.
   She says to him: 'If you give me €n, I will have 3 times as much as you'.
   Given that all these statements are true and that n is a positive integer, what are the possible values for n?

3. Let a, b, c be positive real numbers. Prove that

$$\left(\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\right)^2 \geq (a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right).$$

4. Two equal equilateral triangles, one with red sides and one with blue sides, overlap so that their sides intersect at six points, forming a hexagon. Prove



   (a) the sum of the squares of the lengths of the red sides of the hexagon is equal to the sum of the squares of the lengths of the blue sides of the hexagon;

   (b) the sum of the lengths of the red sides of the hexagon is equal to the sum of the lengths of the blue sides of the hexagon.

5. Prove that, for every positive integer n which ends in the digit 5,

$$20^n + 15^n + 8^n + 6^n$$

is divisible by 1991.

Deadline: Wednesday, 21:00 o'clock