Freitag – Samstag, Klasse 9/10:

International Zhautykov Olympiad 2005, day 1, problem 3:

Let A be a set of 2n points on the plane such that no three points are collinear. Prove that for any distinct two points a, b of A there exists a line that partitions A into two subsets each containing n points and such that a and b lie on different sides of the line.

Freitag – Samstag, Klasse 11/12:

USAMO 2005, problem 5

Let n be an integer greater than 1. Suppose 2n points are given in the plane, no three of which are collinear. Suppose n of the given 2n points are colored blue and the other n colored red. A line in the plane is called a balancing line if it passes through one blue and one red point and, for each side of the line, the number of blue points on that side is equal to the number of red points on the same side.

Prove that there exist at least (a) two
(b) three balancing lines.

Samstag – Sonntag, Klasse 9/10:

USAMO 2000, problem 4

Find the smallest positive integer n such that if n squares of a 1000x1000 chessboard are colored, then there will exist three colored squares whose centers form a right triangle with sides parallel to the edges of the board.

Samstag – Sonntag, Klasse 11/12:

BMO (British Mathematical Olympiad) 2002, 2nd Round, problem 4

Suppose that $B_1, \dots, B_N$ are N spheres of unit radius arranged in space so that each sphere touches exactly two others externally. Let P be a point outside all these spheres, and let the N points of contact be $C_1, \dots, C_N$. The length of the tangent from P to the sphere B_i ($1 \leq i \leq N$) is denoted by t_i. Prove the product of the quantities t_i is not more than the product of the distances PC_i.