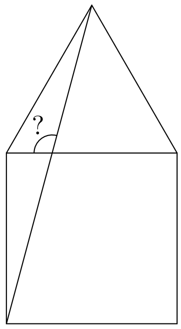
Problem 1. A diagram of a house consists of a square and an equilateral triangle of the same side length. What is the size of the marked angle in degrees?

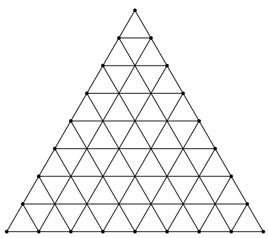


Problem 2. Members of a sport team are posing for a photo. They stand in a row and all of them are wearing team jerseys numbered by some distinct positive integers. The photographer notices that the one standing at the right end of the row has number 72 and that the number of any other team member divides the number of his neighbour on the right (the sides are given from the point of view of the photographer). How many athletes at most could be standing there?

Problem 3. In an ensemble of string players, everyone can play the violin or the viola and exactly one quarter of all the members can play both the instruments. Furthermore, we know that 32 people can play the violin and 23 can play the viola. How many members are there altogether?

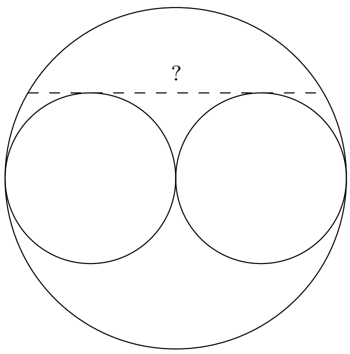
Problem 4. Cecil has multiplied five consecutive positive integers, obtaining number C. David has done the same, but his sequence started with a number one greater than Cecil's, leading to the product D. What was the smallest of the numbers that David multiplied, provided that $C/D = 4/5$?

Problem 5. To each small triangle assign the number of small triangles with which it shares an edge. Determine the sum of all these numbers.



Problem 6. Find the largest positive integer n such that $n^2 - 5n + 6$ is a prime number.

Problem 7. Two circles of radii 1 are touching in the centre of a big circle which is also tangent to the two smaller circles. Determine the length of the dashed segment, which is tangent to the smaller circles and its endpoints lie on the big circle as in the picture.

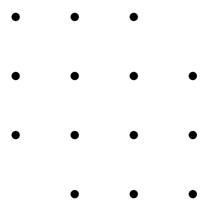


Problem 8. Four mathematicians sat around a table and have ordered a large bowl of pretzels. Daniel left for the toilet. Each minute, Adam, Beatha and Cyril took one pretzel, divided it into three equal pieces and ate it. After some time, Daniel returned to the table and they all continued eating one pretzel each minute, but Daniel got to eat 2/5 of

each pretzel, while the others got to eat 1/5. After some time, Adam noted that Daniel ate exactly the same portion as himself. What is the ratio of time in which Daniel was absent to the time he was present?

Problem 9. An exchange office in Prague offers these coins: 1 Czech crown for 40 cents, 2 crowns for 50 cents, 5 crowns for 1 euro, 10 crowns for 2 euros, 20 crowns for 4.1 euros and 50 crowns for 9.9 euros. Mark wants to exchange all of his 11.8 euros, but he does not want to buy more than one coin of each type. How much will he get (in Czech crowns)? Find the sum of all solutions.

Problem 10. In a regular grid of unit squares these fourteen points are marked. How many rectangles are there having four marked points as their vertices? Recall that square is a special case of rectangle.



Problem 11. What is the value of the positive integer n for which the least common multiple of 60 and n is larger by 777 than the greatest common divisor of 60 and n?

Problem 12. An ant sits in the centre of a face of a regular tetrahedron of edge length 1. By crawling on the surface of the tetrahedron it wants to get to the centre of an edge that does not lie on the same face as the ant sits in. What is the length of the shortest way that it needs to walk to get there?

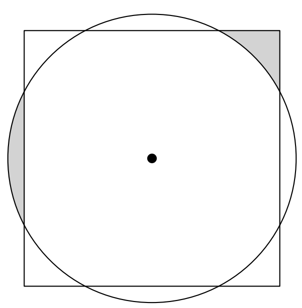
Problem 13. Agnieszka, Brunhilda, Cecilia and Doina drew numbers 3, 6, 9 and 12 without repetitions in some order. (No number was shared by two or more women.) We know that two of them always lie and two say the truth. They said the following:

- Agnieszka: I got twice as much as Doina.
- Brunhilda: I got three times as much as Doina.
- Cecilia: I got four times as much as Doina.
- Doina: I did not get the least.

What is the product of the numbers that the two liars got?

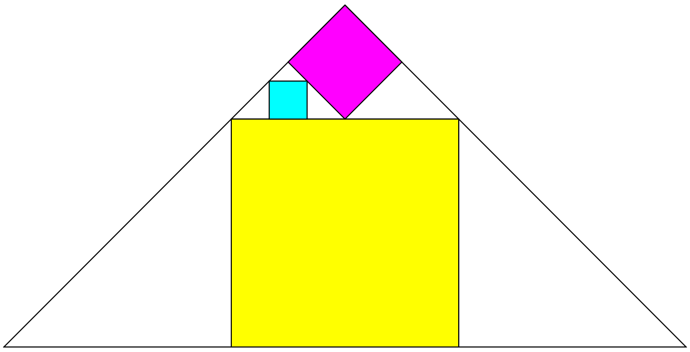
Problem 14. Find the smallest positive integer that has exactly 24 positive divisors and exactly 8 of them are odd.

Problem 15. A circle and a square have the same centre. If the grey regions have the same area, what is the ratio of the side of the square to the radius of the circle?



Problem 16. Lenka wrote the sequence $1, 2, 3, \dots, 20$ and a plus or minus sign between each pair of consecutive numbers in such a way that the resulting sum equalled 192. In how many ways could she have done that?

Problem 17. Three squares have been put into an isosceles right triangle as in the picture:



What fraction of the area of the triangle is taken by the cyan square?

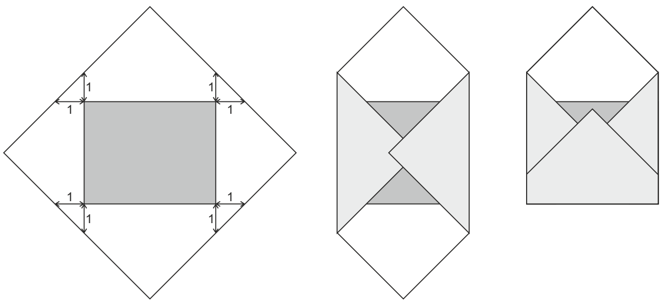
Problem 18. Find the sum of all positive integers which cannot be written as $2a + 3b$ for some coprime positive integers a and b.

Recall that positive integers m and n are coprime if $\gcd(m, n) = 1$.

Problem 19. A polynomial is called heavy if it has two integer roots differing by one, all of its coefficients are integers and their sum equals 2020. How many heavy quadratic polynomials, i.e. expressions of the form $ax^2 + bx + c$, are there?

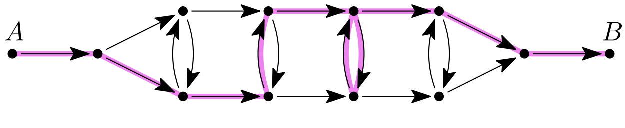
Problem 20. A train consisted of 40 carriages numbered from 1 to 40, each having the capacity of 40 passengers. At the beginning, there was one passenger sitting in carriage 1, two passengers in carriage 2 etc., up to forty passengers in carriage 40. However, for technical reasons, the last carriage had to be removed from the train and all its passengers moved to the carriages with smaller number in such a way that they took the first available seat and if the carriage was already full, they proceeded to check the next one with a smaller number. During the journey, the same thing happened subsequently to carriages $39, 38, \ldots, 23$. How many passengers were in carriage 2 after this had happened?

Problem 21. Jacek cannot buy envelopes, so he is planning to make them by himself. To make a rectangular envelope Jacek is taking a square sheet of paper with diagonal 30 cm, folds the left and the right corner, then folds the bottom corner and closes the envelope by folding the top corner. In order to glue the envelope correctly, there needs to be precisely 1 cm overlap, as in the picture, and after closing the envelope the top corner cannot be below the bottom edge of the envelope. What is the maximal possible width of the envelope?



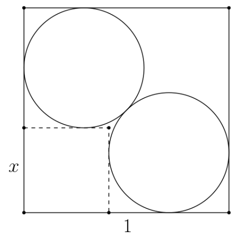
Problem 22. Martha picked three positive integers a, b, c and computed the three sums of pairs $a + b$, $b + c$, $c + a$, obtaining three distinct squares of integers. What was the smallest possible value of $a + b + c$?

Problem 23. In the following diagram, how many paths are there from point A to point B that use each arrow at most once? (One such path is drawn in violet.)



Problem 24. How many four-digit numbers are there with the property that any two neighbouring digits differ exactly by 3? A number may not start with zero.

Problem 25. Each of two circles of equal size touches the other circle and also two sides of a unit square as in the picture. Compute the side length of the dashed square, which shares one corner with the unit square and touches the circles.



Problem 26. Real numbers $x_1, x_2, \ldots, x_{2020}$ have these properties:

- Whenever we sum all of the numbers except a single x_i with i odd, the result is 2.
- Whenever we sum all of the numbers except a single x_i with i even, the result is 0.

What is the value of the sum $x_1 + x_2 + \cdots + x_{2020}$?

Problem 27. Two players are playing a game, alternating moves. Each move consists of changing a positive integer n into another positive integer in the range $[\frac{n}{3}, \frac{n}{2}]$. A player who cannot move loses. For how many starting numbers in the range $[1, 1000]$ is the first player able to win with optimal strategy?

Problem 28. Two circles with diameters $AB = 17$ and $AC = 7$ meet in points A and D. We further know that $CD = 4$. Consider all possible distances of the centres of the circles and compute their product.

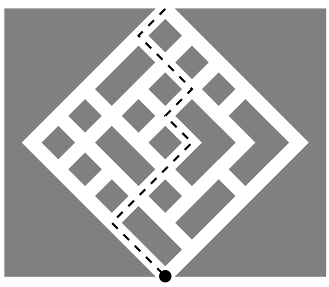
Problem 29. Seven trolleybuses run between fourteen stations on one line. Each trolleybus starts in one of the stations and moves in one direction until it reaches the end of the line, which is either the first or the last station on the line, where it turns around and continues in the other direction. All trolleybuses maintain constant speed and they pass exactly one station in a minute. MSc Birne has placed them in such a way, that

1. there is at most one trolleybus at each station, and
2. there will be at most one trolleybus at each station in one minute, no matter in what directions the trolleybuses go.

In how many ways could that have been done? The trolleybuses are considered identical.

Problem 30. Marian has written a book with 2020 pages labelled $1, 2, 3, \ldots 2020$. After revision, he added an abstract consisting of 11 pages in front of the book. How many digits does he have to rewrite so that every page gets its proper label? A digit from an old page number can only be used in the new one on the same position (ones, tens, hundreds, etc.) and newly written digits, such as the leading 1 appearing in the change $95 \to 106$, are not counted.

Problem 31. A puck is dropped into the top of the plinko box and slides downward until it falls out the bottom. How many different paths through the box are possible? One example path is shown.



Problem 32. The king and his 100 knights sit down at the round table. The vegetarians are served cheese and everyone else is served chicken. But the king has a smaller portion of chicken than the knight to his left, so he orders everyone to pass their plate to the right. Now the king has a reasonable portion of chicken, but 64 knights have the wrong meal, so everyone passes their plate to the right again. Once more the king has a smaller portion of chicken than the knight to his left, so everyone passes their plate to the right a third time. Now only 2 knights (and not the king) have the wrong meal, so they trade places and the feast begins. How many of the king's knights ate chicken?

Problem 33. David would love to draw a triangle ABC and points D, E in the interiors of sides AB and BC respectively in such a way, that triangles ABC, AEC, ADE, BDE were similar. What is the sum of all possible values of the size of angle BAC in degrees?

Problem 34. Five night workers have to schedule their shifts for the following 10 nights so that every night exactly two workers have a shift and those two workers cannot have the shift night after. How many time schedules containing each possible pair of the workers exist?

Problem 35. Lucy has got a triangle with side lengths 32, 50, and x. Furthermore, there is a triangle similar to and having two sides of the same length as Lucy's, but it is not congruent to it. Find the sum of all possible values of x.

Problem 36. A runner is training on a track shaped as the perimeter of a regular 40-gon. His training plan is as follows: first, he runs from the initial vertex to the clockwise-adjacent one and has a short break there. He continues in this manner until he has a break at the initial vertex. Then he starts again, this time having a break after passing every other edge and again continues until a break at the initial vertex where he further increases the length of one sprint by one, etc. How many times will the runner rest before completing a whole loop without a break around the 40-gon? There is no break at the beginning nor after this last run.

Problem 37. A doctoral student living on a cubic planet needs to spend a travelling budget by visiting universities which are located at the vertices of the cube. The budget covers 2020 trips and must be used completely. The student starts at his home university and realises the first trip by moving to one of the neighbouring (by an edge of the cube) universities. He always chooses the next university to visit randomly with the only condition that he cannot return home sooner than by the 2020-th trip. What is the probability that he comes back home by the 2020-th trip?

Problem 38. Let us define $x \star n = (2-x)^n + x^3 - 6x^2 + 12x - 5$ for any real number x and a positive integer n. Determine the sum of all real numbers a solving equation

$$(\dots(a \star 2020) \star 2019) \star \dots \star 2) \star 1 = a.$$

Problem 39. Four friends decided to enrol in some of the four different available courses. They decided that each of them enrols in at least one course and that there will be exactly one course in which more than one of them enrol. In how many ways can they do that?

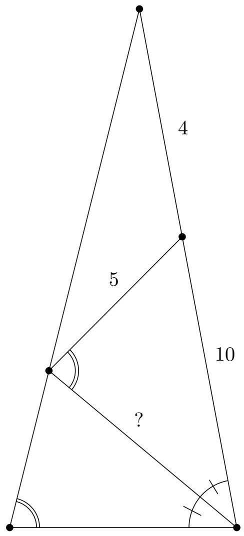
Problem 40. Consider the sum of all numbers having exactly 1000^{1000} digits, all of them being only 1, 2, or 4. What are the last three digits of this sum?

Problem 41. In the triangle ABC, the angle at vertex A is twice as big as the angle at vertex B. All sides are integer valued and the length of the side BC is as small as possible. What is the product of the side lengths of the triangle?

Problem 42. There is a round table with 30 seats. In how many ways can we choose some (at least one) of the seats in such a way that no two neighbouring seats are chosen? Choices which differ by rotation are considered distinct.

Problem 43. Martin attended an online workshop on wire bending and got a homework to manufacture the construction from the picture. He remembered the two marked pairs of equally large angles and lengths of three line segments. Unfortunately, he forgot the fourth one (marked by "?") and thus is now struggling with the homework.

Help him and determine the missing length.

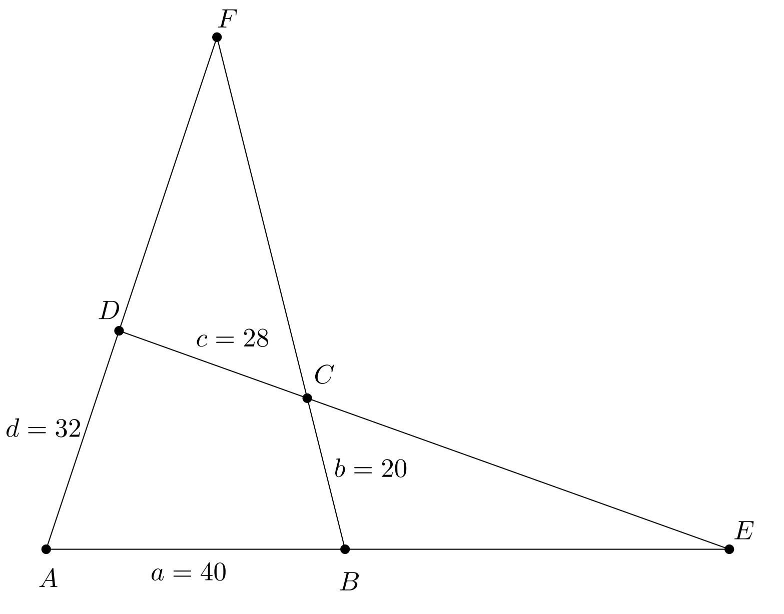


Problem 44. Consider functions $f\colon \mathbb{N} \to \mathbb{N}$ satisfying the condition

$$f(m+n) \geq f(m) + f(f(n)) - 1$$

for all $m, n \in \mathbb{N}$. Find the arithmetic mean of all possible values of $f(2020)$.

Problem 45. Farmer Karl possesses a quadrangular piece of land with the side lengths $a = 40$, $b = 20$, $c = 28$, $d = 32$ as in the picture. By heritage he receives the two triangular pieces of land BEC and DCF, which are delimited by the original sides of his land and their extensions, respectively. If he needs a fence of length 80 for the section $BE + EC$, how long is the fence for the section $CF + FD$?



Problem 46. Determine for how many $k \in \{1, \ldots, 2020\}$ the equation $p^3 + q^3 + r^3 = 3pqr + k$ has a solution (p, q, r) for some positive integers p, q, r.