CONTACT
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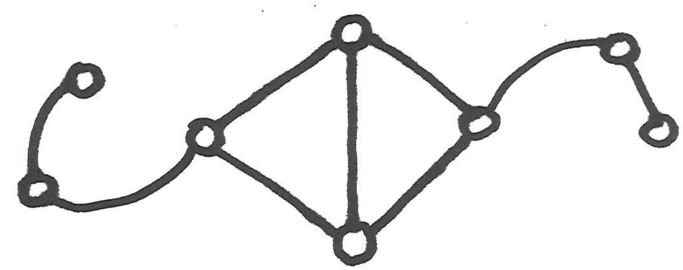
WITH ANY QUESTIONS

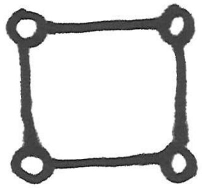
GRAPH THEORY



FOR KIDS!

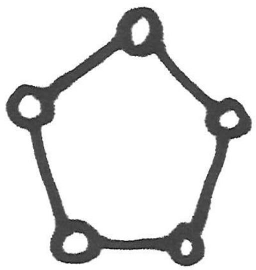
ANOTHER 3D SOLID:

$$\underset{V}{\bigcirc} - \underset{E}{\bigcirc} + \underset{F}{\bigcirc} = \underline{\qquad}$$



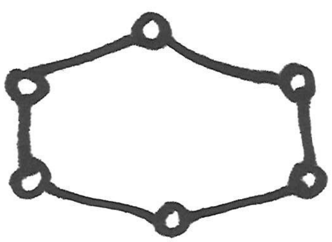
V E R

◯ - ◯ + ◯ = ____



V E R

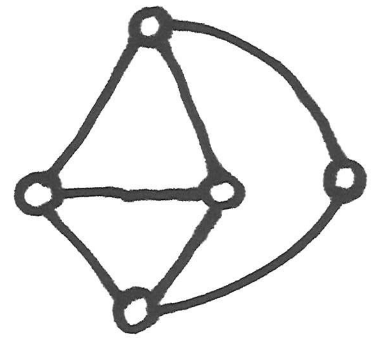
◯ - ◯ + ◯ = ____



V E R

◯ - ◯ + ◯ = ____

A <u>GRAPH</u> IS A COLLECTION OF VERTICES JOINED BY EDGES.



THIS GRAPH HAS

5 VERTICES

7 EDGES

AND IT DIVIDES THE PLANE INTO

4 REGIONS

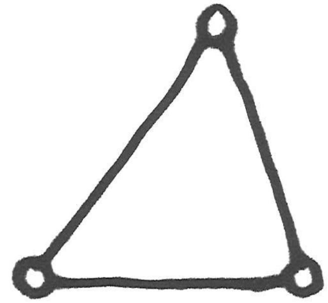
THE MATHEMATICIAN LEONHARD EULER NOTICED SOMETHING PECULIAR WHEN HE CALCULATED:

$$\begin{pmatrix}\text{NUMBER OF}\\ \textbf{VERTICES}\end{pmatrix} - \begin{pmatrix}\text{NUMBER OF}\\ \textbf{EDGES}\end{pmatrix} + \begin{pmatrix}\text{NUMBER OF}\\ \textbf{REGIONS}\end{pmatrix}$$

$$V - E + R$$

THIS NUMBER IS NOW KNOWN AS THE <u>EULER</u> <u>CHARACTERISTIC</u>.

LET'S CALCULATE IT!



VERTICES ____

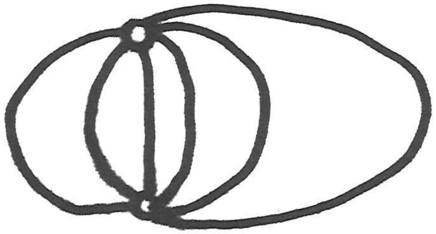
EDGES ____

REGIONS ____

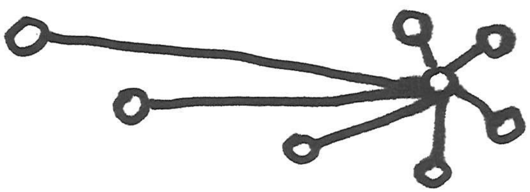
$$\overset{V}{\bigcirc} - \overset{E}{\bigcirc} + \overset{R}{\bigcirc} = \underline{\qquad}$$

DO WE ALWAYS GET 2?

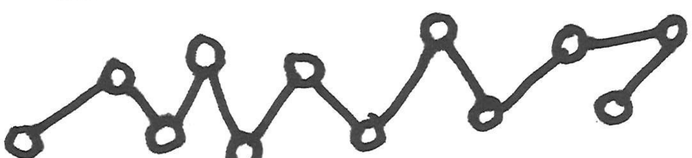
LET'S TRY SOME EXTREME CASES:



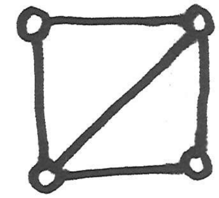
V ○ − E ○ + R ○ = ____



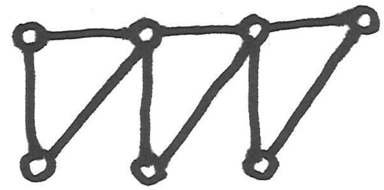
V ○ − E ○ + R ○ = ____



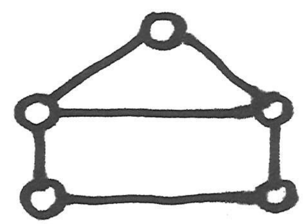
V ○ − E ○ + R ○ = ____



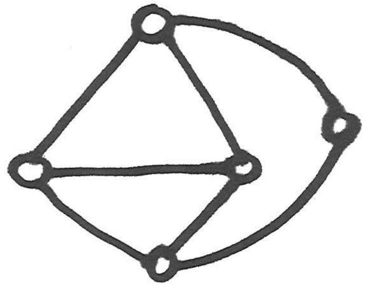
V ○ − E ○ + R ○ = ____

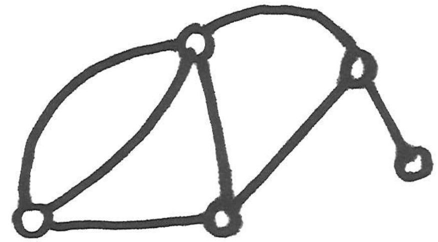


V ○ − E ○ + R ○ = ____



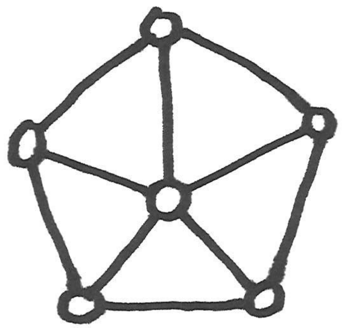
V ○ − E ○ + R ○ = ____

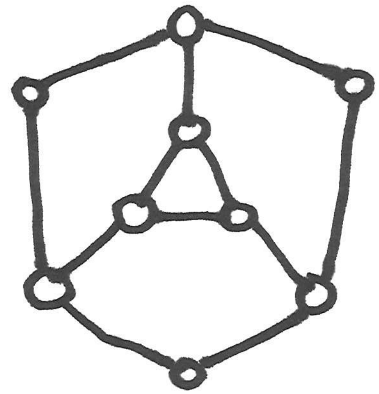




V − E + R = ____

V − E + R = ____

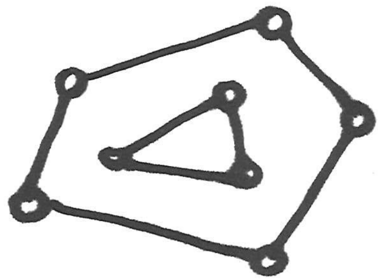




V − E + R = ____

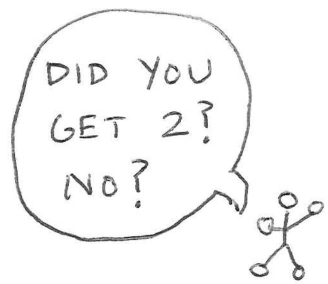
V − E + R = ____

LET'S TRY THIS GRAPH:

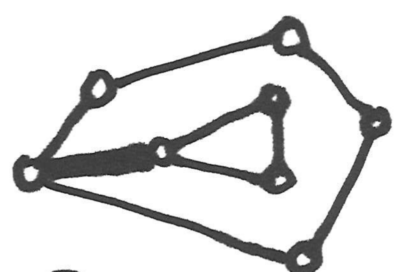


V − E + R = ____

THIS GRAPH IS NOT CONNECTED.



WE CAN CONNECT THE TWO COMPONENTS BY ADDING AN EDGE.



V − E + R = ____

NOW IT WORKS!

TRY IT YOURSELF!

MY GRAPH:

V − E + R = ____

MY GRAPH:

$$\overset{V}{\bigcirc} - \overset{E}{\bigcirc} + \overset{R}{\bigcirc} = \_\_\_\_$$

MY GRAPH:

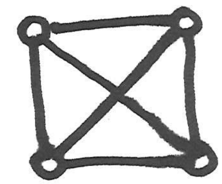
$$\overset{V}{\bigcirc} - \overset{E}{\bigcirc} + \overset{R}{\bigcirc} = \_\_\_\_$$

CONCLUSION:

EVERY CONNECTED PLANAR GRAPH HAS

$$V - E + R = 2.$$

HOW ABOUT THIS GRAPH?

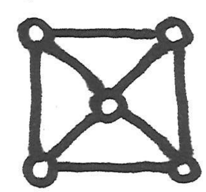


V − E + R = ___

DID IT WORK?

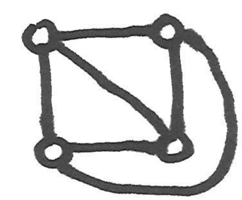
IN THIS GRAPH, THE DIAGONAL EDGES CROSS. LET'S FIX THAT.

ADD A VERTEX



V − E + R = ___

MOVE AN EDGE



V − E + R = ___

NOW IT WORKS!

A <u>PLANAR</u> GRAPH CAN BE DRAWN WITHOUT EDGES CROSSING.

FOR A CONNECTED PLANAR GRAPH, IS THE EULER CHARACTERISTIC ALWAYS 2?

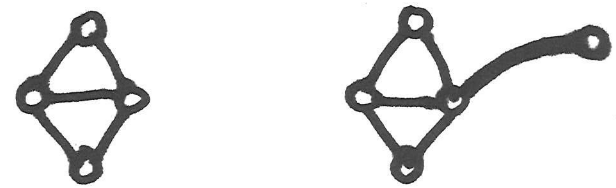
YES!

- IT STARTS OUT TRUE

ONE VERTEX
NO EDGES
ONE REGION

$$\begin{array}{ccccccc} V & & E & & R & & \\ 1 & - & 0 & + & 1 & = & \underline{2} \end{array}$$

- IT STAYS TRUE WHEN WE ADD A CONNECTED VERTEX.



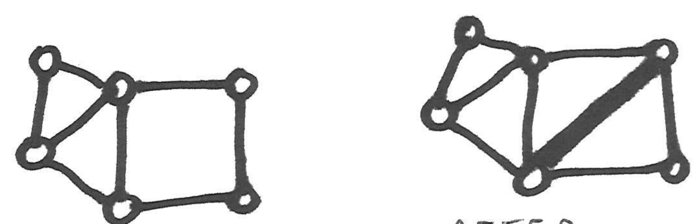
BEFORE AFTER

ONE NEW VERTEX
ONE NEW EDGE

$$V - E + R$$

VERTICES & EDGES BALANCE EACH OTHER

- IT STAYS TRUE WHEN WE CUT A REGION WITH A NEW EDGE.



BEFORE AFTER

ONE NEW EDGE
ONE EXTRA REGION

$$V - E + R$$

EDGES & REGIONS BALANCE

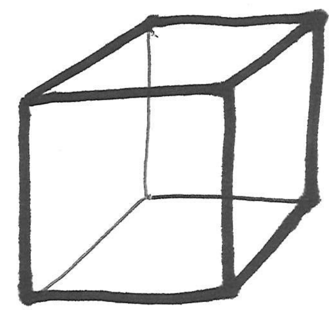
CAN YOU DRAW A 3D SOLID?

MY SOLID:

V - E + F = ___

LET'S CONSIDER THE SURFACES OF SOME THREE-DIMENSIONAL SOLIDS.

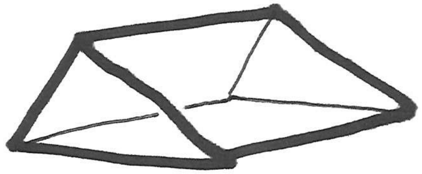
CUBE



VERTICES - EDGES + FACES = ___

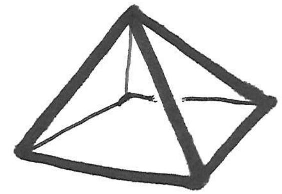
WE CALL THEM "FACES" INSTEAD OF "REGIONS".

TRIANGULAR PRISM



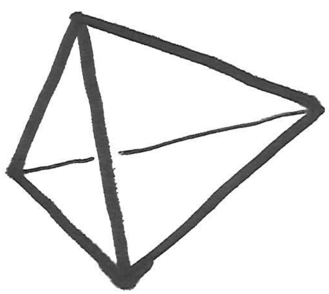
V − E + F = ___

PYRAMID (SQUARE BASE)



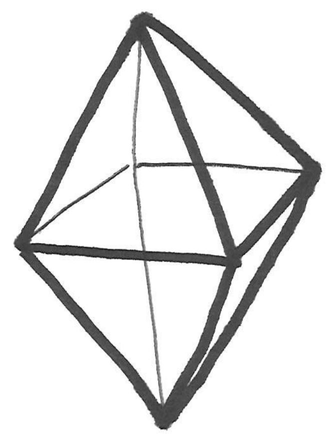
V − E + F = ___

TETRAHEDRON



V − E + F = ___

OCTAHEDRON



V − E + F = ___